Vacuum Properties of a Non-Local Thirring-Like Model

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Abstract

We use path-integral methods to analyze the vacuum properties of a recently proposed extension of the Thirring model in which the interaction between fermionic currents is non-local. We calculate the exact ground state wave functional of the model for any bilocal potential, and also study its long-distance behavior. We show that the ground state wave functional has a general factored Jastrow form. We also find that it posess an interesting symmetry involving the interchange of density-density and current-current interactions.

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1 Introduction

In the last years there has been much interest in the study of low-dimensional field theories. One of the main reasons for this revival can be found in striking achievements of the material sciences that have allowed to build ultranarrow semiconductor structures [1] in which the motion of the electrons is confined to one dimension [2]. One important tool for the theoretical understanding of the one-dimensional (1d) electron system is the Tomonaga-Luttinger (TL) model [3] [4] [5], which can be considered as the paradigm of Luttinger Liquid (LL) behavior [6] [2]. This model describes a non-relativistic gas of massless particles (the electrons) with linear free dispersion relation and two-body, forward-scattering interactions. In a recent work [7], a non-local Thirring model (NLT) with fermionic currents coupled by general (symmetric) bilocal potentials was presented, which contains the TL system as a special case. In that work, the complete non-local bosonization of the model was presented, calculating in particular the dispersion relations of the relevant bosonic modes involved in the system.

In quantum field theory, the wave functional is not the most usual quantity to calculate. The reason is that, for the evaluation of scattering data, the relevant quantities are Green functions. Nevertheless, with the application of quantum field techniques to model condensed matter systems, it becomes useful to get the information contained in the ground state of the system.

In this paper we address our attention to the vacuum properties of the above mentioned model. As it is well-known, ground-state wave functionals (GSWF's) have in general very complex structures. Due to this fact, their universal behavior has been seldom explored in the past. Fortunately, in a recent series of papers, an alternative way to compute GSWF's was presented [8] [9] [10]. By conveniently combining the operational and functional approaches to quantum field theories, these authors provided a systematic path-integral method that, at least in the context of $1+1$ systems, seems to be more practical than the previously known semiclassical [11] and Bethe ansatz [12] [13] techniques. We take advantage of these advances and apply them to shed some light on the vacuum structure of the NLT. This is a relevant issue for several reasons. On the one hand, our work can be viewed as an extension of the path-integral approach to wave functionals to the case in which non-local interactions are taken into account. On the other hand our studies clarify the physical content of a model that is interesting by its own right, due to its direct connection with many-body systems.

The paper is organized as follows. In Section 2 we define the NLT model and recall the steps that enable to obtain its path-integral bosonization. In Section 3 we describe the density representation of wave functionals and combine the results of [7] and [10] in order to evaluate the GSWF for the NLT. We get a closed formula that gives the probability of the vacuum state as a functional, not only of the density

configuration but also of the potentials that bind the original fermionic particles of the system. This result allows us to find a non-trivial symmetry of this vacuum with respect to the interchange of potentials. We also discuss the general electromagnetic response of the model. In Section 4 we analyze the long-distance behavior of the GSWF for the NLT. Exploiting the generality of this model we specialize the results of the previous Section to some particular potentials. This permits us to make contact with the TL [3] [4] [5] and Sutherland [13] models. In Section 5 we sketch the Grassmann representation of wave functionals and show how to implement it in the present context. Although the mathematical structure for the GSWF is more involved in this representation, once again we obtain a closed expression for the probability of the vacuum as a functional of both Grassmann sources and potentials. Finally, in Section 6 we summarize our main results and conclusions.

2 The Model and the Non-Local Bosonization Method

We start defining our model by writing the partition function

$$Z = \int D\bar{\Psi}\ D\Psi\ e^{-S}, \tag{1}$$

where the action S can be split as

$$S = S_0 + S_{int}, \tag{2}$$

with

$$S_0 = \int d^2x\ \bar{\Psi} i\partial\!\!\!/ \Psi \tag{3}$$

and

$$S_{int} = -\frac{g^2}{2}\int d^2x d^2y\ [V_{(0)}(x,y)J_0(x)J_0(y) + V_{(1)}(x,y)J_1(x)J_1(y)] \tag{4}$$

where the electron field Ψ is written as

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix},$$

with Ψ_1 (Ψ_2) describing right (left) movers.

Concerning the electronic kinetic energy, we have set the Fermi velocity equal to 1. The interaction piece of the action has been written in terms of currents J_μ defined as

$$J_\mu \ = \ \bar{\Psi}\gamma_\mu\Psi, \tag{5}$$

$V_{(\mu)}(x,y)$ are symmetric bilocal arbitrary potentials describing the electron-electron (e-e) interactions. The model is not relativistic except for the special case $V_{(0)} = V_{(1)}$.

Our first goal is to express the functional integral (1) in terms of fermionic determinants. To this end we define new currents

$$K_\mu(x) = \int d^2y \; V_{(\mu)}(x,y) J_\mu(y). \tag{6}$$

(Note that no sum over repeated indices is implied when a subindex (μ) is involved).

The usual procedure in order to match the quartic interaction between fermions consists in introducing auxiliary fields A_μ so that one can write

$$\begin{aligned} Z &= \int D\bar{\Psi} \; D\Psi \; e^{-S_0} \\ &\int \quad DA_\mu \; \delta[A_\mu - K_\mu] exp[\int d^2x \; J_\mu A_\mu] \end{aligned} \tag{7}$$

On the other hand, we represent the δ functionals as integrals of exponentials over new fields $C_\mu(x)$, thus obtaining

$$\begin{aligned} Z &= \int D\bar{\Psi} \; D\Psi \; DA_\mu \; e^{-S_0} \\ &\quad exp \int d^2x \; J_\mu A_\mu \int DC_\mu exp[-\int d^2x \; (A_\mu - K_\mu) C_\mu] \end{aligned} \tag{8}$$

At this point one sees that the fermionic piece of the action (the free part and the terms involving the currents J and K) can be cast in a local form by defining the "potential transformed" fields

$$\bar{C}_\mu(x) = \int d^2y \; V_{(\mu)}(x,y) C_\mu(y), \tag{9}$$

We then get

$$\begin{aligned} Z &= \int D\bar{\Psi} \; D\Psi \; DA \; D\bar{C} \\ &\quad exp\{-\int d^2x \; [\bar{\Psi}(i\partial\!\!\!/ + (A\!\!\!/ + \bar{C}\!\!\!/))\Psi + \\ &+ \quad C_\mu(x) A_\mu(x)]\}. \end{aligned} \tag{10}$$

This equation, in turn, suggests the following change of variables

$$\begin{aligned} A_\mu + \bar{C}_\mu &= \tilde{A}_\mu, \\ A_\mu - \bar{C}_\mu &= \tilde{C}_\mu, \end{aligned} \tag{11}$$

giving

$$Z = \int D\tilde{A}\ D\tilde{C}\ det(i\partial\!\!\!/ + \tilde{A}\!\!\!/) \\ exp \int d^2x\ d^2y\ \{-\frac{b_{(\mu)}(x,y)}{4}[\tilde{A}_\mu(x) - \tilde{C}_\mu(x)]\tilde{C}_\mu(y)\}, \tag{12}$$

where we have defined the inverse potentials b through the identities

$$\int V_{(\mu)}(x,y)b_{(\mu)}(x,z)d^2x\ = \delta^{(2)}(y-z), \tag{13}$$

In the above expression for Z one sees that, by virtue of the change of variables (11), the fields $\tilde{C}$ play no direct role in the fermionic determinant. They are actually artefacts of our method, whereas the fields A describe the physically relevant bosonic degrees of freedom. Therefore, the next step is to perform the integrals in $\tilde{C}$. This can be easily done, as usual, by conveniently shifting the fields. When this is done one finds a field that describes negative metric states. In order to agree with Klaiber's operational prescription we absorb the decoupled ghost partition function in the overall normalization constant [7]. Taking these considerations into account, and setting from now on $\tilde{A} = A$, one finally gets

$$Z = \int DA\ e^{-S'[A]} det(i\partial\!\!\!/ + A\!\!\!/). \tag{14}$$

with

$$S'[A] = \int d^2x\ d^2y \frac{1}{2}[A_\mu(x)b_{(\mu)}(x,y)A_\mu(y)]. \tag{15}$$

Thus we have been able to express the partition function for the NLT in terms of a fermionic determinant. This is a necessary condition to apply the path-integral approach to non-local bosonization which, combined with the methods of [8] and [10], will enable us tu derive the GSWF for the NLT. An interesting point of our approach is that one can go further quite a long way without specifying the potentials. This will be shown in the next Sections, where we shall undertake the evaluation of the GSWF in the density and Grassmann representations.

3 Ground State Wave Functional in the Density Representation

Generally, the wave function can be labeled by the eigenvalues of the particle number operator. In the case of an N-particle system, $\psi(x_1,\ldots,x_n) =< \psi|x_1,\ldots,x_n >$, where $|x_1,\ldots,x_n >$ is an eingenstate of the particle density operator $\hat{\rho}(x) = \hat{c}^\dagger(x)\hat{c}(x)$ with eigenvalue $\rho(x) = \sum_{i=1}^{N}\delta(x-x_i)$. This representation is called the density representation and we can label it by $\psi(\rho) =< \psi|\rho >$.

In the case of a dense system, $\rho(x)$ is a general distribution (not necessarily δ's) and $\psi(\rho)$ is a functional of a density rather than a function.

In a fermionic system, antisymmetrization of the wave function is supposed. In spite of the fact that $\rho(x)$ is an even distribution in a dense system, we will see that the Pauli exclusion principle is still satisfied.

This section is devoted to the calculation of the GSWF in the density representation, for the non-local Thirring model, described in the previous Section.

The GSWF is related with the equal-time density correlation function $< \rho(p)\rho(-p) >$. In references [8] and [10], it was shown an interesting relation between the GSWF in the density representation and the generating functional $Z(Q)$ (see Appendix):

$$|\psi_0[\rho]|^2 = \int DQ_0 e^{-i\int d\mathbf{x}Q_0(\mathbf{x})\rho(\mathbf{x})} \lim_{Q_0(x)\to Q_0(\mathbf{x})\delta(x_0)} Z(Q_0, Q_1 = 0) \tag{16}$$

So, the first step towards the calculation of $|\psi_0[\rho]|^2$ is to evaluate $Z(Q_\mu)$. To this aim, we take advantage of equation (14), where the generating functional was written in terms of a fermionic determinant. Coupling the system minimally to an external gauge potential $Q_\mu(x)$, we obtain from (14):

$$\begin{aligned} Z(Q_\mu) &= \int DA\ Det\,\{i\ \partial\!\!\!/ + g\ A\!\!\!/\} \times \\ &\times\ \exp\left\{-\frac{1}{2}\int d^2x d^2y\ (A_\mu(x) + \frac{1}{g}Q_\mu(x))b_{(\mu)}(x-y)(A_\mu(y) + \frac{1}{g}Q_\mu(y))\right\} \end{aligned} \tag{17}$$

To calculate the fermionic determinant, we make a chiral change of variables in the fermionic fields

$$\begin{aligned} \psi(x) &= e^{-g[\gamma_5\phi(x)+i\eta(x)]}\chi \\ \bar{\psi}(x) &= \bar{\chi}e^{-g[\gamma_5\phi(x)-i\eta(x)]} \end{aligned} \tag{18}$$

With this change of variables the measure transforms as

$$D\bar{\psi}D\psi\ = J_F[\phi,\eta]D\bar{\chi}D\chi \tag{19}$$

It is also possible (in 1+1 dimensions), to split the gauge field in a longitudinal plus a transversal component in the following way

$$A_\mu(x) = \epsilon_{\mu\nu}\partial_\nu\phi + \partial_\mu\eta \tag{20}$$

These changes lead to

$$Det\left\{i\ \not\partial + g\ \not A\right\} = J_F[\phi,\eta] Det\left\{i\ \not\partial\right\} \tag{21}$$

As it is well-known (see for instance [14]) the Jacobian associated to this change of variables is:

$$\log J_F[\phi,\eta] = \frac{g^2+\alpha}{2\pi}\int d^2x\ \phi\Box\phi \tag{22}$$

where α is an arbitrary parameter that can be fixed with gauge invariance arguments.

Putting all this together, we finally find a bozonized generating functional

$$Z(Q_\mu) = \int D\phi D\eta e^{-S_{eff}(\phi,\eta,Q_\mu)} \tag{23}$$

where

$$\begin{aligned}
S_{eff} &= \frac{g^2+\alpha}{2\pi}\int d^2x\ (\partial_\mu\phi)^2 \\
&+ \int d^2xd^2y\left\{b_{(0)}(x-y)\partial_1\phi(x)\partial_1\phi(y) + b_{(1)}(x-y)\partial_0\phi(x)\partial_0\phi(y)\right\} \\
&+ \int d^2xd^2y\ \left\{b_{(0)}(x-y)\partial_0\eta(x)\partial_1\phi(y) - b_{(1)}(x-y)\partial_1\eta(x)\partial_0\phi(y)\right\} \\
&- \frac{1}{g}\int d^2xd^2y\ \phi(x)(\epsilon_{\mu\nu}\partial_\nu b_{(\mu)}(x-y))Q_\mu(y) \\
&- \frac{1}{g}\int d^2xd^2y\ \eta(x)(\partial_\mu b_{(\mu)}(x-y))Q_\mu(y) \\
&+ \frac{1}{2g^2}\int d^2xd^2y\ Q_\mu(x)b_{(\mu)}(x-y))Q_\mu(y)
\end{aligned} \tag{24}$$

It is simpler to evaluate the generating functional in momentum space. To do this, we Fourier transform eq. (24) obtaining:

$$\begin{aligned}
S_{eff} &= \int\frac{d^2p}{(2\pi)^2}\ [\tilde\phi(p)\tilde\phi(-p)A(p) + \tilde\eta(p)\tilde\eta(-p)B(p) + \tilde\phi(p)\tilde\eta(-p)C(p)] + \\
&+ \frac{i}{g}\int\frac{d^2p}{(2\pi)^2}\ \left\{\tilde\phi(p)(\epsilon_{\mu\nu}p_\nu\hat b_{(\mu)}(p)\tilde Q_\mu(-p)) + \tilde\eta(p)(p_\mu\hat b_{(\mu)}(p))\tilde Q_\mu(-p))\right\} \\
&+ \frac{1}{2g^2}\int\frac{d^2p}{(2\pi)^2}\tilde Q_\mu(p))\hat b_{(\mu)}(p)\tilde Q_\mu(-p)
\end{aligned} \tag{25}$$

where

$$A(p) = \frac{g^2+\alpha}{2\pi}\, p^2 + \frac{1}{2}[\hat{b}_{(0)}(p)p_1^2 + \hat{b}_{(1)}(p)p_0^2], \tag{26}$$

$$B(p) = \frac{1}{2}[\hat{b}_{(0)}(p)p_0^2 + \hat{b}_{(1)}(p)p_1^2], \tag{27}$$

$$C(p) = [\hat{b}_{(0)}(p) - \hat{b}_{(1)}(p)]p_0 p_1. \tag{28}$$

To integrate (25), we first decouple the fields $\tilde{\phi}$ and $\tilde{\eta}$ by means of

$$\tilde{\phi} = \hat{\xi} - \frac{C}{2A}\hat{\zeta} \tag{29}$$

$$\tilde{\eta} = \hat{\zeta} \tag{30}$$

and then, we integrate the quadratic integrals in $\hat{\xi}$ and $\hat{\zeta}$. We thus obtain:

$$Z(Q_\mu) = \exp\left\{-\int \frac{d^2p}{(2\pi)^2}\, \tilde{Q}_\mu(p)\pi_{\mu\nu}(p)\tilde{Q}_\nu(-p)\right\} \tag{31}$$

where

$$\pi_{\mu\nu} = \frac{1}{2\pi}\frac{p_0^2+\mathbf{p}^2}{\{\frac{g^2}{\pi}v_0(p)+1\}\mathbf{p}^2 + \{\frac{g^2}{\pi}v_1(p)+1\}p_0^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \tag{32}$$

with $v(p) = \mathcal{F}(V(x-x'))$.

Note that, for the generating functional $Z(Q_\mu)$ being gauge invariant, the polarization tensor $\pi_{\mu\nu}$ must be transversal ($p_\mu \pi_{\mu\nu} = 0$). This property is automatically satisfied for any potential due to the tensor structure of (32).

For the present model being well defined, the potentials must satisfy the following (sufficient) condition:

$$\left(\frac{g^2}{\pi}v_0(p)+1\right)\left(\frac{g^2}{\pi}v_1(p)+1\right) > 0 \tag{33}$$

If this relation is not satisfied, the *euclidean* $\pi_{\mu\nu}$ may have a pole in the real p_0 axis. This corresponds to the propagation of a runaway mode, breaking the unitarity of the model. Roughly speaking, relation (33) means that the density-density interaction and the current-current interaction must be both repulsive or both attractive (note that it is a sufficient condition, not necessary). As a by product we have obtained *the exact electromagnetic response of the system for any potential* v_0 *or* v_1. For example, if we apply an arbitrary electric field to the fermionic system, it will induce an electric current and a charge density given by (in Minkovsky space):

$$\rho(p) \equiv < \bar{\psi}\gamma^0\psi > \tag{34}$$

$$= -\frac{i}{8\pi^3}\frac{\mathbf{p}}{\{\frac{g^2}{\pi}v_0(p)+1\}\mathbf{p}^2 - \{\frac{g^2}{\pi}v_1(p)+1\}p_0^2} \times \mathbf{E}(p_0,\mathbf{p})$$

$$J(p) \equiv < \bar{\psi}\gamma^1\psi > \tag{35}$$
$$= \frac{i}{8\pi^3}\frac{p_0}{\{\frac{g^2}{\pi}v_0(p)+1\}\mathbf{p}^2 - \{\frac{g^2}{\pi}v_1(p)+1\}p_0^2} \times \mathbf{E}(p_0,\mathbf{p})$$

where $\mathbf{E} = i(\mathbf{p}Q_0 - p_0Q_1)$ is the applied electric field.

Equation (35) implies that for instantaneous potentials, the system allows the propagation of free waves with dispersion relation

$$\omega_0 = \pm\sqrt{\frac{\frac{g^2}{\pi}v_0(\mathbf{p})+1}{\frac{g^2}{\pi}v_1(\mathbf{p})+1}}|\mathbf{p}| \tag{36}$$

These waves are related with the propagation of the bosonic modes $\hat{\xi}$ and $\hat{\zeta}$ (see eq. (29) and (30)) discussed in ref. [7]. We shall return to this issue in the next section, when we consider the Tomonaga-Luttinger model. In particular, note that in the special case $v_0(\mathbf{p}) = v_1(\mathbf{p})$, the dispersion relation is the free one $\omega_0 = \pm|\mathbf{p}|$ ($v_f = 1$).

After this digression, let us now face the evaluation of the GSWF. Fourier transforming equation (31) and using (16) we have (in euclidean space):

$$|\psi_0[\rho]|^2 = \int D\tilde{Q}_0 e^{-\frac{1}{2\pi}\int d\mathbf{p}\tilde{Q}_0(\mathbf{p})\rho(\mathbf{p})} e^{-\frac{1}{(2\pi)^2}\int d\mathbf{p}\ \tilde{Q}_0(\mathbf{p})\bar{\Pi}_{00}(\mathbf{p})\tilde{Q}_0(-\mathbf{p})} \tag{37}$$

where

$$\bar{\pi}_{00}(\mathbf{p}) = \int dp_0\ \pi_{00}(p_0,\mathbf{p}) \tag{38}$$

It is a simple task to integrate equation (37) obtaining

$$|\psi_0[\rho]|^2 = e^{\frac{1}{4}\int d\mathbf{p}\rho(\mathbf{p})(\bar{\pi}_{00})^{-1}\rho(-\mathbf{p})} \tag{39}$$

with $\bar{\pi}_{00}$ given by (38).

This result express the probability (not the amplitude) of a particular density distribution to be realized in the ground state of the non-local Thirring model.

We can go further if we suppose that the potentials are local in time (as in any non-relativistic model). In this case, v_0 and v_1 are p_0-independent and we can integrate (38) explicitely. So for any instantaneous potential we have

$$\bar{\pi}_{00}(\mathbf{p}) = \int_{-\infty}^{+\infty}\frac{dp_0}{2\pi}\ \frac{\mathbf{p}}{(\frac{g^2}{\pi}v_0(\mathbf{p})+1)\mathbf{p}^2 + (\frac{g^2}{\pi}v_1(\mathbf{p})+1)p_0^2}$$
$$= \frac{\pi}{2}\sqrt{\frac{1}{(\frac{g^2}{\pi}v_0(\mathbf{p})+1)(\frac{g^2}{\pi}v_1(\mathbf{p})+1)}}|\mathbf{p}| \tag{40}$$

In this way, from (40) and (39) we obtain

$$|\psi_0[\rho]|^2 = e^{\frac{1}{2}\int d\mathbf{p}\rho(\mathbf{p})\sqrt{(\frac{g^2}{\pi}v_0(\mathbf{p})+1)(\frac{g^2}{\pi}v_1(\mathbf{p})+1)}(\frac{1}{|\mathbf{p}|})\rho(-\mathbf{p})} \tag{41}$$

This equation is the main result of this section, and gives the *exact ground state wave functional for the non-local Thirring model* defined by eqs. (3) and (4).

Let us point out some general features of this wave functional. From (41) we see that the vacuum of the theory has the non-trivial symmetry

$$V_{(0)}(x-y) \longleftrightarrow V_{(1)}(x-y) \tag{42}$$

This symmetry tells us that the density-density interaction is completely equivalent to the current-current interaction, provided we are studying only vacuum properties. It is clear that it should be broken by the excited states of the spectrum since the action (4) is not symmetric.

Another property of (41), is that $|\psi[\rho]|^2$ in the N- particle subspace, has a general factored Jastrow form:

$$|\psi(x_1,\ldots,x_n)|^2 = \prod_{i,j}\left[\varphi(|x_i-x_j|)\right]^{\lambda} \tag{43}$$

To see this more clearly, let us rewrite (41) in configuration space,

$$|\psi_0[\rho]|^2 = e^{\pi\int dx_1\ dx_2\rho(x_1)f(|x_1-x_2|)\rho(x_2)} \tag{44}$$

with

$$f(|x_1-x_2|) = \int \frac{dp}{2\pi}\,\sqrt{(\frac{g^2}{\pi}v_0(\mathbf{p})+1)(\frac{g^2}{\pi}v_1(\mathbf{p})+1)}\left(\frac{1}{|\mathbf{p}|}\right)e^{i\mathbf{p}(x_1-x_2)} \tag{45}$$

We now consider the Fock subspace with fixed n particles and n holes (antiparticles), since we are working with $<\rho>=0$, i. e. , without chemical potential. In this circumstance one has

$$\rho(x) = \sum_{i=1}^{n}\left(\delta(x-x_i)-\delta(x-y_i)\right) \tag{46}$$

where $\{x_i\}(\{y_i\})$ is the position of the particles (holes).

Replacing (46) in (44) we have

$$|\psi(x_1,\ldots,x_n,y_1,\ldots,y_n)|^2 = \prod_{i<j}e^{2\pi f(|x_i-x_j|)}e^{2\pi f(|y_i-y_j|)}\prod_{i,j}e^{-2\pi f(|x_i-y_j|)} \tag{47}$$

that is the general factored form for our model. Clearly, the complexity of this expression depends on the form of the potentials. In the next section we will study universal properties of the long-distance behavior. But, in order to gain confidence

and to somehow test our result, let us analyze the simpler example of a local Thirring model, i.e. $v_0 = v_1 = 1$. We then get

$$f(|x|) = (\frac{g^2}{\pi} + 1)\mathcal{F}(|\mathbf{p}|^{-1}) = \frac{1}{\pi}(\frac{g^2}{\pi} + 1)\ln(|x|) \tag{48}$$

Inserting this expression in (47) we obtain

$$|\psi(x_1, \ldots, x_n, y_1, \ldots, y_n)|^2 = \frac{\prod_{i<j} |x_i - x_j|^{2\mu} |y_i - y_j|^{2\mu}}{\prod_{i,j} |x_i - y_j|^{2\mu}} \tag{49}$$

with $\mu = g^2/\pi + 1$, which is the correct result [10].

4 Long distance behavior and the connection with the Tomonaga-Luttinger and Sutherland models

In this section we analyze the general long-distance behaviour of the GSWF. We also specialize the non-local Thirring model for particular potentials, showing that it contains the Tomonaga-Luttinger [3] [5] and the Sutherland [13] models as particular cases.

We have already shown, that in the limit $V_0 = V_1 = \delta^2(x - y)$, our model reproduces the local Thirring model. Let us now consider two non-local potentials of the form:

$$V_0 \propto |x - y|^{\alpha} \delta(x_0 - y_0) \tag{50}$$
$$V_1 \propto |x - y|^{\beta} \delta(x_0 - y_0) \tag{51}$$

Their fourier transforms are [15]

$$v_0(\mathbf{p}) \propto -2\sin(\frac{\alpha}{2}\pi)\Gamma(\alpha + 1)|\mathbf{p}|^{-\alpha-1} \tag{52}$$
$$v_1(\mathbf{p}) \propto -2\sin(\frac{\beta}{2}\pi)\Gamma(\beta + 1)|\mathbf{p}|^{-\beta-1} \tag{53}$$

The long distance behavior of $|\psi(\rho)|^2$, is dominated by $\lim_{x\to\infty} f(x)$, or equivalenty $\lim_{\mathbf{p}\to 0} \tilde{f}(\mathbf{p})$(see eqs. (44) and (45)). This behavior depends on the different values of α and β.

For large x (small $\mathbf{p}$) we have the following situation,

$$\left.\begin{array}{l}\alpha>-1\\ \beta>-1\end{array}\right\}\rightarrow\ \tilde{f}(\mathbf{p})\propto|\mathbf{p}|^{-\frac{\alpha+\beta+2}{2}}\ \rightarrow f(x)\propto-\text{cte.}|x|^{\frac{\alpha+\beta}{2}}$$
$$\left.\begin{array}{l}\alpha>-1\\ \beta<-1\end{array}\right\}\rightarrow\ \tilde{f}(\mathbf{p})\propto|\mathbf{p}|^{-\frac{\alpha+3}{2}}\ \rightarrow f(x)\propto-\text{cte.}|x|^{\frac{\alpha+1}{2}}$$
$$\left.\begin{array}{l}\alpha<-1\\ \beta>-1\end{array}\right\}\rightarrow\ \tilde{f}(\mathbf{p})\propto|\mathbf{p}|^{-\frac{\beta+3}{2}}\ \rightarrow f(x)\propto-\text{cte.}|x|^{\frac{\beta+1}{2}}$$
$$\left.\begin{array}{l}\alpha<-1\\ \beta<-1\end{array}\right\}\rightarrow\ \tilde{f}(\mathbf{p})\propto\frac{1}{|\mathbf{p}|}\ \rightarrow f(x)\propto\ \text{cte.}\ln|x| \tag{54}$$

We note that the Jastrow form (43) is kept for all potentials at long distances, but the physics may be very different depending on the values of α anb β. From the first line of (54), we see that for $\alpha>-1$ and $\beta>-1$ we have two types of long distance limits. If $\alpha+\beta<0$, the wave functional tends asymptotically to a constant, a typical behavior of an incompressible fluid. But, if $\alpha+\beta>0$, the wave functional goes to zero exponentially as $\exp(-\text{cte.}|x-x'|^{\frac{\alpha+\beta}{2}})$, characterizing a confining phase. Lines two and three of (54) also characterize a confining phase, because this exponential decay of the wave function implies that asymptotic fermionic states cannot exist. Last but not least, the case $\alpha<-1,\ \beta<-1$, represent another phase, that can be identified as the Thirring phase, since the general form of the wave function is the same as in the local Thirring model with the appropriate redefinition of the coupling constant (the exponent in the Jastrow wave function).

Another way of classifying these phases, is through the transport properties of the systems. That is to say, each phase is associated to a free propagating mode with different dispersion relations in each phase. From (36) and (53), we can write the long distance dispersion relations for the free propagation modes as:

$$\left.\begin{array}{l}\alpha>-1\\ \beta>-1\end{array}\right\}\rightarrow\ w_0=\pm|\mathbf{p}|^{\frac{\beta-\alpha+2}{2}}$$
$$\left.\begin{array}{l}\alpha>-1\\ \beta<-1\end{array}\right\}\rightarrow\ w_0=\pm|\mathbf{p}|^{\frac{1-\alpha}{2}}$$
$$\left.\begin{array}{l}\alpha<-1\\ \beta>-1\end{array}\right\}\rightarrow\ w_0=\pm|\mathbf{p}|^{\frac{\beta+3}{2}}$$
$$\left.\begin{array}{l}\alpha<-1\\ \beta<-1\end{array}\right\}\rightarrow\ w_0=\pm|\mathbf{p}| \tag{55}$$

Note that these relations distinguish between V_0 and V_1 (α and β). This is so because, in order to propagate such modes, not only the ground state is necessary, but the excited states are necessary also.

This completes our analysis of the long distance behavior in our non-local Thirring model. Let us now show, by specifying the potentials, that this general theory con-

tains some other models already discussed in the literature related to 1-d strong correlated systems.

We shall apply the approach developed in previous Sections to the Tomonaga-Luttinger model [3] [4] [5]. This model describes a non-relativistic gas of spinless and massless particles (electrons) in which the dispersion relation is taken to be linear. The free-particle Hamiltonian is given by

$$H_0 = v_F \int dx \Psi^\dagger(x)(\sigma_3 p - p_F)\Psi(x) \tag{56}$$

where v_F and p_F are the Fermi velocity and momentum respectively ($v_F p_F$ is a convenient origin for the energy scale). σ_3 is a Pauli matrix and Ψ is a column bispinor with components Ψ_1 and Ψ_2 ($\Psi^\dagger = (\Psi_1^\dagger \;\; \Psi_2^\dagger)$). The function $\Psi_1(x)$ [$\Psi_2(x)$] is associated with the motion of particles in the positive [negative] x direction. The interaction piece of the Hamiltonian, when only forward scattering is considered, is

$$H_{int} = \int dx \int dy \sum_{a,b} \Psi_a^\dagger(x)\Psi_a(x)V_{ab}(x,y)\Psi_b^\dagger(y)\Psi_b(y) \tag{57}$$

where $a, b = 1, 2$, and the interaction matrix is parametrized in the form

$$V_{ab} = \begin{pmatrix} v_1 & v_2 \\ v_2 & v_1 \end{pmatrix}. \tag{58}$$

Using the imaginary-time formalism one can show that the finite-temperature [16] [17] action for this problem becomes

$$\begin{aligned} S_{TL} &= \int_0^\beta d\tau \int dx \, \{p_0\gamma_0(\partial_\tau - v_p p_F)\Psi + v_F \mathbf{p}\gamma_1 \partial_x \Psi\} \\ &+ \int_0^\beta d\tau \int dx \int dy \sum_{a,b} \Psi_a^\dagger \Psi_a(x,\tau)V_{ab}(x,y)\Psi_b^\dagger \Psi_b(y,\tau). \end{aligned} \tag{59}$$

For simplicity, we shall set $v_F = 1$ and consider the case $v_1 = v_2$ in (58) [5]. We shall also restrict ourselves to the zero temperature limit ($\beta \to \infty$). Under these conditions it is easy to verify that S_{TL} coincides with the non-local Thirring model discussed in the precedent Sections, provided that the following identities hold:

$$\begin{aligned} g^2 &= 2 \\ V_{(0)}(x,y) &= v_1(x,y) = v_2(x,y) = v(x_1 - y_1)\delta(x_0 - y_0) \\ V_{(1)} &= 0 \end{aligned} \tag{60}$$

Of course one has also to make the shift $\mathbf{p}\gamma_0\partial_0\Psi \to \mathbf{p}\gamma_0(\partial_0 - p_F)\Psi$ and identify $x_0 = \tau$, $x_1 = x$.

One then can employ the method described in the preceding sections in order to study the Tomonaga-Luttinger model. In [7] this model was studied with emphasis in the bosonization approach. It has also been previously studied, through a different functional approach, by D.K. Lee and Y. Chen [18]. These authors, however, avoided the use of the decoupling technique applied here. Now we want to examine the vacuum properties of the model and show how to evaluate the GSWF considering the model as a special case of a non-local Thirring model.

Let us first focus our attention to the dispersion relations corresponding to the elementary excitations of the model at hand. This dispersion relation is, of course, a special case of (36). Using (60) we have

$$\omega_-^2(\mathbf{p}) = \mathbf{p}^2\{1 + \frac{2v(\mathbf{p})}{\pi}\} \tag{61}$$

which is the well-known result for the spectrum of the charge-density excitations of the TL model in the Mattis-Lieb version [5].

We can now compute the corresponding GSWF by replacing (60) in (41):

$$|\psi_0[\rho]|^2 = e^{\frac{1}{2}\int d\mathbf{p}\rho(\mathbf{p})\sqrt{(\frac{2}{\pi}v(\mathbf{p})+1)}(\frac{1}{|\mathbf{p}|})\rho(-\mathbf{p})} \tag{62}$$

For example for $V(x) = 1/|x|^2$ we can deduce, employing the same analysis used for the general model, that the long distance behavior of the wave functional is essentialy of the Thirring type, with a renormalization of the exponent of the Jastrow form. On the other hand, if we consider the $3d$ Coulomb potential $V(\mathbf{p}) = 1/|\mathbf{p}|^2$, then, $|\psi|^2 \propto exp\{-\text{cte}|x - x'|\}$, showing again the landmark of a confining phase. This means, that no asymptotic fermionic states can exist, and the dispersion relation (61) refers to the "condensed" bosonic degrees of freedom. Note that this is a *relativistic bosonic mass mode* ($\omega_-^2(\mathbf{p}) = \mathbf{p}^2 + \frac{2}{\pi}$).

An interesting observation is the following. If we change in (60) $V_0 \leftrightarrow V_1$, we obtain another model with $j - j$ interaction rather than $\rho - \rho$ interaction. This new model has the same vacuum properties of the former (the same GSWF), implying that for the "Coulomb $j-j$ interaction" we have also a confining behavior. However, the confined bosonic modes are very different, since their dispersion relation is now $\omega_0^2 = \mathbf{p}^4/(\mathbf{p}^2 + \frac{2}{\pi})$.

Another special case that we can analyze is the Sutherland's model [13]. It is a system of non-relativistic spinless fermions interacting via a pair interaction potential $V(|x - y|)$ whose hamilonian is:

$$H = \int dx\ \frac{1}{2m}|\partial_x\psi(x)|^2 + \frac{1}{2}\int dx \int dy(\psi^\dagger\psi)(x)V(|x-y|)(\psi^\dagger\psi)(y) \tag{63}$$

The spectrum of this hamiltonian was calculated exactly in [13].

It can be shown that at long distance, the Sutherland's model (63) is equivalent to the model described by the action

$$\begin{aligned} S_0 &= \int d^2x\ [\bar{\Psi} i\partial\!\!\!/ \Psi - \frac{g^2}{2}(\bar{\psi}\gamma_\mu\psi)^2] \\ &+ \frac{1}{2}\int dxdy\ (\psi^\dagger\psi)(x)u(|x-y|)(\psi^\dagger\psi)(y) \end{aligned} \tag{64}$$

It is easy to realize that this action is a special case of the one described by (3) and (4), provided we identify

$$V_0(x-y) = \delta(x-y) - \frac{1}{g^2}u(x-y)\delta(x_0-y_0) \tag{65}$$

$$V_1(x-y) = \delta(x-y) \tag{66}$$

Replacing the Fourier transform of this expressions in (41) we immediately arrive at

$$|\psi_0|^2 = e^{\frac{1}{2}(\frac{g^2}{\pi}+1)\int d\mathbf{p}\ \rho(\mathbf{p})\sqrt{1-\frac{\tilde{u}(\mathbf{p})}{\pi(1+g^2/\pi)}}(\frac{1}{|\mathbf{p}|})\rho(-\mathbf{p})} \tag{67}$$

This wave functional was extensively studied in [10].

5 GSWF in the Grassmann representation

Another representation for the wave functional is the so called *Grassmann representation* [8] [9] [10], in which the vacuum is projected onto fermionic coherent states. This representation allows to implement the antisymmetry of the wave functional automatically. However, the final expression for the functional is less intuitive than the density representation.

In a subspace with a finite number of particles, we can build a fermionic coherent state by

$$|\xi_1,\ldots,\xi_n> = e^{\sum_{j=1}^n \xi_j C^\dagger(x_j)}|0> \tag{68}$$

where ξ_j are Grassmann variables and $C^\dagger(x_j)$ are the fermionic creation operators.

The wave function is constructed by projecting this states onto the vacuum, i. e. :

$$\psi(\xi_1,\ldots,\xi_n) = <0|\xi_1,\ldots,\xi_n> = <0|e^{\sum_{j=1}^n \xi_j C^\dagger(x_j)}|0> \tag{69}$$

We can see the relation between equation (69) and the *orbital* wave functions by expanding the exponential,

$$\psi(\xi_1,\ldots,\xi_n) = \sum_{n=1}^{N}\frac{1}{(n!)^2}\left(\prod_{i=1}^{n}\xi_i\right)\psi(x_1,\ldots,x_n) \tag{70}$$

where $\psi(x_1,\ldots,x_n) =< 0|C^\dagger(x_1),\ldots,C^\dagger(x_n)|0 >$. So, the *orbital* wave functions are the coefficients of a polynomial expansion in the Grassmann variables.

In the case of a dense system, a coherent state is given by

$$|\chi >= e^{i\int dx \;\; \chi(x)\hat{\bar{\psi}}(x)+\hat{\psi}(x)\bar{\chi}(x)}|0 > \tag{71}$$

where $\chi(x)$ is a Grassmann field, and a wave functional is labeled by

$$\psi(\chi,\bar{\chi}) =< 0|e^{i\int dx \;\; \chi(x)\hat{\bar{\psi}}(x)+\hat{\psi}(x)\bar{\chi}(x)}|0 > \tag{72}$$

Let us now derive the wave functional (72) for the NLT. The Grassmann representation of the wave functions can be built in a way that is quite similar to the one discussed in Section 3 for the density representation (see Appendix).

It can be shown that the probability for the state χ to occur in the ground state is given by

$$|\psi_0[\bar{\chi},\chi]|^2 = \int D\bar{\eta}\; D\eta\; Z[\bar{\eta},\eta]\; exp(-i\int dx\; (\bar{\chi}\eta+\bar{\eta}\chi)) \tag{73}$$

where

$$Z[\bar{\eta},\eta] = \int D\bar{\Psi}\; D\Psi\; e^{-S_0} exp[\int d^2x\; (\frac{g^2}{2}J_\mu K_\mu - \bar{\eta}\Psi - \bar{\Psi}\eta)] \tag{74}$$

and we have taken the equal-time limit $\chi(x,t) = \chi(x)\delta(t)$, and similarly for $\bar{\chi}$. S_0, J_μ and K_μ were defined in Section 2, and η and $\bar{\eta}$ are, of course, a couple of fermionic sources.

One can rewrite $Z[\bar{\eta},\eta]$, by using the procedure depicted in Section 2, based on the introduction of a set of auxiliary vector fields. This leads to an expression which is nothing but the generalization of equation (14) for non-vanishing fermionic sources:

$$Z[\bar{\eta},\eta] = \int DA\; e^{-S'[A]}\; exp[-\int d^2x\bar{\Psi}(i\partial\!\!\!/ + A\!\!\!/)\Psi]\; exp[-\int dx(\bar{\eta}\Psi+\bar{\Psi}\eta)] \tag{75}$$

where $S'[A]$ is given in (15).

Performing now a uniform translation in the fields Ψ and $\bar{\Psi}$, one gets

$$Z[\bar{\eta},\eta] = \int DA\; e^{-S'[A]}\; det(i\partial\!\!\!/ + A\!\!\!/)\; exp-[\int dxdy\; \bar{\eta}(x)\; (i\partial\!\!\!/ + A\!\!\!/)^{-1}(x,y)\; \eta(y)] \tag{76}$$

As explained in Section 3, we can make a chiral transformation in the fermionic measure and express A_μ in terms of two scalar fields Φ and ω (see equations (18) and (20)). Taking into account the corresponding Jacobian, we obtain

$$Z[\bar{\eta},\eta] = \int D\Phi\ D\omega\ e^{-S_{eff}[\Phi,\omega]}\ \exp[-\int dxdy\ \bar{\eta}(x)\ G_F[\Phi,\omega]\ \eta(y)] \tag{77}$$

where $S_{eff}[\Phi,\omega]$ picks up the contribution of the Jacobian, and coincides with (24) if one sets $Q_\mu = 0$. We have also defined

$$G_F[\Phi,\omega] = e^{-g[\gamma_5\phi(x)+i\eta(x)]}\ (i\partial\!\!\!/)^{-1}(x,y)\ e^{-g[\gamma_5\phi(y)-i\eta(y)]} \tag{78}$$

At this stage we are ready to insert (77) in the expression for the GSWF, equation (73). In so doing one sees that the integration in the fields η and $\bar{\eta}$ is elementary, yielding

$$|\psi_0[\bar{\chi},\chi]|^2 = \int D\Phi\ D\omega\ e^{-S_{eff}[\Phi,\omega]}\ \exp\left[-\int dxdy\ \bar{\chi}(x)\ {G_F}^{-1}[\Phi,\omega]\ \chi(y)\right] \tag{79}$$

In the functional integrand of (79) we have omitted a factor $det\ G_F[\Phi,\omega]$, which can be shown to be constant, using, for instance, a coherent-state definition of the functional integral.

Now one can expand the exponential and perform the integrations over Φ and ω for each term of the series. The result can be written as

$$|\psi_0[\bar{\chi},\chi]|^2 = \sum_n\ \left(\frac{2i}{\pi}\right)^n\ \frac{1}{n!^2}\ \int\left(\prod_{i=1}^n dx_i dy_i\right)\prod_{j=1}^n \bar{\chi}_{\alpha_j}(x_j)\ \chi_{\beta_j}(y_j)\ F\left([x_j,y_j]\right)_{\alpha_k\beta_k} \tag{80}$$

where the indices α_i and β_i indicate Dirac spinor components and $j,k = 1,...,n$.

The function $F\left([x_j,y_j]\right)_{\alpha_k\beta_k}$ is the product of two factors:

$$F_0\left([x_j,y_j]\right)_{\alpha_k\beta_k} = (det\frac{1}{(x_i-y_j)})\prod_{i=1}^n (\gamma_1)_{\alpha_i\beta_i} \tag{81}$$

which comes from the contribution of the free fermion (equal-time) propagators, and a bosonic factor that corresponds to a multipoint (equal-time) correlation function of vertex operators,

$$\mathcal{B}(x_j,y_j) = \left\langle \exp\left[\sum_{j=1}^n (s_j\Phi(x_j,t) + t_j\Phi(y_j,t))\right]\ \exp\left[-i\sum_{j=1}^n(\omega(x_j,t)-\omega(y_j,t))\right]\right\rangle \tag{82}$$

where s_i $(t_i) = 1$ or -1 if α_i $(\beta_i) = 1$ or 2. Up to this point the results of this Section are formally equal to those obtained for the local Thirring model, in ref.[10]. However, we have to stress that in our case the vacuum expectation value in (82) is to be computed for the *non-local* model defined by $S_{eff}[\Phi,\omega]$. Introducing the

Fourier transformed fields $\tilde{\Phi}(p)$ and $\tilde{\omega}(p)$, it is straightforward to express the bosonic factor as

$$\mathcal{B}(x_j, y_j) = \int D\tilde{\Phi}\ D\tilde{\omega}\ \exp\left[-\left(S_{eff}[\tilde{\Phi}, \tilde{\omega}] + \int d^2p\ (\tilde{\Phi}(p)J(\mathbf{p}) + \tilde{\omega}(p)K(\mathbf{p}))\right)\right] \tag{83}$$

with

$$J(\mathbf{p}) = -\sum_{j=1}^{n}\left(s_j\ e^{i\mathbf{p}x_j} + t_j\ e^{i\mathbf{p}y_j}\right) \tag{84}$$

and

$$K(\mathbf{p}) = i\sum_{j=1}^{n}\left(e^{i\mathbf{p}x_j} - e^{i\mathbf{p}y_j}\right) \tag{85}$$

As usual, the functional integrations in (83) can be easily done just by conveniently shifting the fields. Indeed, if we introduce two new fields $\tilde{\phi}$ and $\tilde{\rho}$ such that

$$\tilde{\Phi}(p) = \tilde{\phi}(p) + M(p) \tag{86}$$

$$\tilde{\omega}(p) = \tilde{\rho}(p) + N(p), \tag{87}$$

the choice

$$M(-p) = \frac{2B(p)J(\mathbf{p}) - C(p)K(\mathbf{p})}{\Delta(p)} \tag{88}$$

$$N(-p) = \frac{2A(p)K(\mathbf{p}) - C(p)J(\mathbf{p})}{\Delta(p)} \tag{89}$$

with A,B and C defined in (25) and $\Delta = C^2 - 4AB$, allows to obtain

$$\mathcal{B} = \exp\left[\frac{-1}{(2\pi)^2}\int d^2p\ \frac{1}{\Delta(p)}\left(B(p)J(\mathbf{p})J(-\mathbf{p}) + A(p)K(\mathbf{p})K(-\mathbf{p}) - C(p)J(\mathbf{p})K(-\mathbf{p})\right)\right] \tag{90}$$

Please note that in this equation we have omitted the explicit dependence on the spatial coordinates, which enters the game through J and K (see eqs.(84) and (85)). After replacing the corresponding expressions the final result can be written in the form

$$\begin{aligned}\mathcal{B} &= \exp\left\{(-1/(2\pi)^2)\int d^2p\ (1/\Delta(p))\sum_{j,k}\left[(s_js_kB(p) - A(p) + is_jC(p))e^{i\mathbf{p}(x_j-x_k)}\right.\right.\\ &+ (t_jt_kB(p) - A(p) - it_jC(p))e^{i\mathbf{p}(y_j-y_k)} + 2(s_jt_kB(p) + A(p))\cos[\mathbf{p}(x_j-y_k)]\\ &+ \left.\left. iC(p)(t_je^{i\mathbf{p}(y_j-x_k)} - s_je^{i\mathbf{p}(x_j-y_k)})\right]\right\}\end{aligned} \tag{91}$$

This result is to be multiplied by the fermionic factor $F_0\left([x_j, y_j]\right)_{\alpha_k \beta_k}$ (see eq.(81)) in order to find the general term in the series of the squared vacuum functional, given by eq.(80). Thus, we have obtained the general structure of the ground-state, not only as a functional of the Grassmann sources ($\bar{\chi}$,χ) but also of the potentials that bind the original fermions of the model. Remember that these potentials are contained in the coefficients A, B and C. If one considers the local limit, which corresponds to contact interactions in coordinate space, one can then easily show that eq.(91), and therefore also the general term in (80), acquire the Jastrow form, i.e. a factorized structure with constant exponents given by simple combinations of A,B and C.

6 Summary and conclusions

In this article we have considered a recently proposed non-local version of the Thirring model, in which densities and currents are coupled by arbitrary potential functions $V_0(|x-y|)$ and $V_1(|x-y|)$, respectively. One of the interesting aspects of this theory is that it describes, as particular cases, relevant many-body systems such as the TL and Sutherland's models.

We have focused our attention on the vacuum properties of this model. In particular, we computed the exact ground-state wave functions, as functionals of external sources and two-body potentials, in both the density (Section 3) and Grassmann (Section 5) representations. In the context of the more intuitive density representation we have stressed several physical features of the model which, in our framework, can be easily discussed. For example we got the exact electromagnetic response of the system for any potential. Concerning the vacuum in itself, we found a non-trivial symmetry with respect to the interchange of density-density and current-current potentials. Of course, this symmetry does not persist at the level of the dispersion relations of the collective modes (plasmons), to which the excited states are expected to contribute. The universal factorized Jastrow form was also obtained. On the other hand, we have analyzed the long-distance behavior of the GSWF (Section 4) for this general model, showing that it contains the TL and Sutherland's models as special cases. In particular we have examined the asymptotic behavior of GSWF's and density-waves frequencies for a wide variety of power-law potentials. This allowed us to identify different phases contained in the non-local Thirring model.

We want to emphasize that our results are valid for abitrary bilocal potentials. This means that in our approach one does not need to specify the couplings in order to get closed formulae for the GSWF's. Therefore these formulae could be directly used to obtain the effect of specific potentials on the vacuum structure. It is also interesting to point out that the techniques we presented could be easily modified in order to study the response of the ground-state in the presence of impurities, following, for example, the lines of ref.[19].

A Path integral approach to wave functionals

The path integral approach to wave functionals was fully developed in refs. [8], [9] and [10].

In order to make this paper self-contained we sketch in this appendix the main steps that enable to deduce equations (16) and (73).

A.1 Density Representation

In any system with particle number conservation we have

$$\partial_t \hat{\rho}(x,t) + \partial_x \hat{j}(x,t) = 0 \tag{1}$$

where ρ (j) is the charge (current) density, and a non-relativistic model should satisfy the following relations:

$$[\hat{\rho}(x), \hat{j}(x')] = -i\partial_x(\delta(x-x')\hat{\rho}(x)), \tag{2}$$

$$[\hat{\rho}(x), \hat{\rho}(x')] = [\hat{j}(x), \hat{j}(x')] = 0. \tag{3}$$

Thus, we can label the quantum states with the eigenvalues of the operator $\hat{\rho}$, and represent $\hat{j}$ by

$$\hat{j}(x)|\psi[\rho]> \equiv -i\rho(x)\partial_x\left(\frac{\delta}{\delta\rho(x)}|\psi[\rho]>\right) \tag{4}$$

This is called the density representation.

Since this states completely expand all the Hilbert space, we can resolve the identity operator as

$$\hat{I} = \int \mathcal{D}\rho \; |[\rho]><[\rho]| \tag{5}$$

Using this identity, it is easy to derive the relation between wave functionals and the functional generator of density correlation functions

$$\begin{aligned} Z(Q_0, Q_1 = 0) &= <0|e^{i\int dx \; Q_0(x)\hat{j}_0(x)}|0> \\ &= \int \mathcal{D}\rho' <0|e^{i\int d^2x \; Q_0(x)\hat{j}_0(x)}|\rho'><\rho'|0> \\ &= \int \mathcal{D}\rho' e^{i\int d^2x \; Q_0(x)\rho'(x)} <0|\rho'><\rho'|0> \\ &= \int \mathcal{D}\rho' e^{i\int d^2x \; Q_0(x)\rho'(x)}|\psi[\rho']|^2 \end{aligned} \tag{6}$$

Taking the limit for fixed time generators we have

$$\lim_{Q_0(x)\to Q_0(\mathbf{x})\delta(x_0)} Z(Q_0, Q_1 = 0) = \int \mathcal{D}\rho' e^{i\int d\mathbf{x} \; Q_0(\mathbf{x})\rho'(\mathbf{x})}|\psi[\rho']|^2 \tag{7}$$

Fourier transforming this expression we finally get

$$\begin{aligned}\int DQ_0 e^{-i\int d\mathbf{x} Q_0(\mathbf{x})\rho(\mathbf{x})} \lim_{Q_0(x)\to Q_0(\mathbf{x})\delta(x_0)} Z(Q_0, Q_1=0) &= \int \mathcal{D}\rho'\delta(\rho'(\mathbf{x})-\rho(\mathbf{x}))|\psi[\rho']|^2 \\ &= |\psi[\rho]|^2 \end{aligned} \tag{8}$$

which is the equation (16).

A.2 Grassmann Representation

Let us begin by considering the generating functional for equal time fermionic correlation functions

$$\begin{aligned} Z(\eta,\bar\eta) &= \int \mathcal{D}\psi\mathcal{D}\bar\psi\ e^{-iS(\bar\psi,\psi)+i\int dx\ \bar\eta\psi+\eta\bar\psi} \\ &= <0|e^{i\int dx\ \bar\eta\hat\psi+\eta\hat{\bar\psi}}|0> \end{aligned} \tag{9}$$

where $\eta(x,x_0)\equiv\eta(x)\delta(x_0)$ is the fermionic source.

The Hilbert space is built through fermionic coherent states:

$$|\chi,\bar\chi>= e^{i\int dx\quad \chi(x)\hat{\bar\psi}(x)+\hat\psi(x)\bar\chi(x)}|0> \tag{10}$$

This space is a complete one, so we can represent the identity operator as

$$I=\int \mathcal{D}\chi\mathcal{D}\bar\chi\ \ |\bar\chi,\chi><\chi,\bar\chi| \tag{11}$$

Inserting this expression in (9) we have

$$Z(\eta,\bar\eta)=\int \mathcal{D}\chi\mathcal{D}\bar\chi <0|e^{i\int dx\ \bar\eta\hat\psi+\eta\hat{\bar\psi}}|\bar\chi,\chi><\chi,\bar\chi|0> \tag{12}$$

The fermionic operators $\hat\psi$ and $\hat{\bar\psi}$ satisfy the following anti-commutation relations

$$\{\hat\psi_i(x),\hat{\bar\psi}_j(y)\}=(\gamma_0)_{ij}\delta(x-y) \tag{13}$$

One can represent these operators by using multiplicative and derivative operators χ_i and $\frac{\delta}{\delta\chi_j}$ acting on the coherent states (10):

$$\begin{aligned}\hat{\bar\psi}_i &= \frac{1}{\sqrt{2}}\left(\bar\chi_i+(\gamma_0)_{ij}\frac{\delta}{\delta\chi_j}\right) \\ \hat\psi_i &= \frac{1}{\sqrt{2}}\left(\chi_i+(\gamma_0)_{ij}\frac{\delta}{\delta\bar\chi_j}\right)\end{aligned} \tag{14}$$

With this operations we obtain

$$Z(\eta,\bar{\eta}) = \int \mathcal{D}\chi\mathcal{D}\bar{\chi} e^{i\int dx\ \bar{\eta}\chi+\eta\bar{\chi}} |\psi_0(\chi+\gamma_0\eta, \bar{\chi}+\gamma_0\bar{\eta})|^2 \tag{15}$$

and making the Grassmann translation

$$\begin{aligned} \chi &\longrightarrow \chi - \gamma_0\eta \\ \bar{\chi} &\longrightarrow \bar{\chi} - \gamma_0\bar{\eta} \end{aligned}$$

we get

$$Z(\eta,\bar{\eta}) = \int \mathcal{D}\chi\mathcal{D}\bar{\chi} e^{i\int dx\ \bar{\eta}\chi+\eta\bar{\chi}} |\psi_0(\chi, \bar{\chi})|^2 \tag{16}$$

Note that formally, (16) is the *fermionic Fourier Transform* of $|\psi_0(\chi,\bar{\chi})|^2$. We can anti-transform this expression obtaining finally

$$|\psi_0[\bar{\chi},\chi]|^2 = \int D\bar{\eta}\ D\eta\ Z[\bar{\eta},\eta]\ exp(-i\int dx\ (\bar{\chi}\eta+\bar{\eta}\chi)) \tag{17}$$

which is the equation (73).

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